STATIONARY FLOW OF COMPOSITE CAPILLARY JETS

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Stationary axisymmetric flow of spiral capillary jets, consisting of immiscible liquids, is studied. Composite capillary jets are used in a number of technological processes. An example is the production of heat-insulating mineral wool by the centrifugal-roller method, in which the forming jets of melt are coated with a layer of bonding material. The practical applications of flows of composite jets has motivated the experimental and theoretical study of such jets [1, 2]. In this work a numerical method of collocation is employed to calculate the flow on the starting section of jet formation [3-6].

1. A cylindrical coordinate system r, θ, z whose z axis is oriented along the symmetry axis of the jet and whose origin is positioned at the center of the output opening is introduced. The radius of the output opening R_* and the velocity $U_* = Q/(\pi R_*^2)$ (Q is the volume flow rate of the jet) are chosen as the characteristic quantities. In the boundary-layer approximation the stationary axisymmetric jet flow is described by the following system of equations and boundary conditions [2, 3]:

$$\frac{\partial U_i}{\partial x}+\frac{\partial V_i}{\partial y}+\frac{V_i}{y}=0,\quad i=1,2; \tag{1.1}$$

$$U_i\frac{\partial U_i}{\partial x}+V_i\frac{\partial U_i}{\partial y}=\frac{1}{\mathrm{Fr}}+\frac{\alpha_i}{\mathrm{Re}}\left(\frac{\partial^2 U_i}{\partial y^2}+\frac{1}{y}\frac{\partial U_i}{\partial y}\right)+F_i,\quad i=1,2,$$
$$F_1=\frac{1}{\mathrm{We}}\left(\frac{1}{H_1^2}\frac{dH_1}{dx}+\frac{\gamma}{H_2^2}\frac{dH_2}{dx}\right),\quad F_2=\frac{\gamma}{\lambda \mathrm{We} H_2^2}\frac{dH_2}{dx}; \tag{1.2}$$

$$U_i\frac{\partial W_i}{\partial x}+V_i\frac{\partial W_i}{\partial y}+\frac{V_iW_i}{y}=\frac{\alpha_i}{\mathrm{Re}}\left(\frac{\partial^2 W_i}{\partial y^2}+\frac{1}{y}\frac{\partial W_i}{\partial y}-\frac{W_i}{y^2}\right),\quad i=1,2; \tag{1.3}$$

$$y=0:\ \frac{\partial U_1}{\partial y}=V_1=W_1=0; \tag{1.4}$$

$$y=H_1:\ U_1\frac{dH_1}{dx}=V_1,\ U_1=U_2,\ V_1=V_2,\ W_1=W_2,\ \frac{\partial U_1}{\partial y}=\alpha\lambda\frac{\partial U_2}{\partial y},$$
$$\frac{\partial W_1}{\partial y}-\frac{W_1}{H_1}=\alpha\lambda\left(\frac{\partial W_2}{\partial y}-\frac{W_2}{H_1}\right); \tag{1.5}$$

$$y=H_2:\ U_2\frac{dH_2}{dx}=V_2,\ \frac{\partial U_2}{\partial y}=\frac{\partial W_2}{\partial y}-\frac{W_2}{H_2}=0, \tag{1.6}$$

where $x = z/R_*$; $y = r/R_*$; H_1 and H_2 are the radii of the interface and the surface of the jet, respectively; U_i, V_i, W_i are the axial, radial, and azimuthal velocity components, where the index i = 1 corresponds to the interior liquid and i = 2 corresponds to the exterior liquid; $\alpha_1 = 1$; $\alpha_2 = \alpha$. The dimensionless parameters are $\alpha = \nu_2/\nu_1$, $\lambda = \rho_2/\rho_1$, $\gamma = \sigma_2/\sigma_1$ (ν_1, ν_2, ρ_1, and ρ_2 are the kinematic coefficients of viscosity and density of the internal and external liquids; σ_1 and σ_2 are the surface tension at the interface and at the surface of the jet) and $\mathrm{Re} = U_*R_*/\nu_1$, $\mathrm{We} = \rho_1U_*^2R_*/\sigma_1$, $\mathrm{Fr} = U_*^2/(gR_*)$ (g is the acceleration of gravity). Here (1.1) is the equation of continuity; (1.2) and (1.3) are the equations of motion for the axial and azimuthal velocity components; (1.4) expresses the regularity of the solution on the axis of the jet; (1.5) and (1.6) include the kinematic relations and the conditions of continuity of the tangential stresses at the interface and on the surface of the jet as well as the equality of the velocity components at $y = H_1$. It is assumed that the pressure distribution in the transverse section is constant for each liquid; this condition holds in the case when the azimuthal velocity is low compared with the axial velocity. The choice of characteristic quantities ensures unit dimensionless flow rate of the jet.

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TABLE 1

Number of calculation	$Q \cdot 10^6$, m^3/sec	$R_* \cdot 10^4$, m	U_*, m/sec	Re	We	Fr	ε_2	t_1	t_2	h	x_a
1	0,0628	1	2	200	20	∞	3	0	0	0,5	40
2	0,0628	0,5	8	400	160	∞	3	0	0	0,5	80
3	0,0628	0,2	50	1000	2500	∞	3	0	0	0,5	200
4	0,0628	0,5	8	400	160	∞	2	0	0	0,5	85
5	0,0628	0,5	8	400	160	∞	3	0	—0,05	0,5	100
6	0,0628	0,5	8	400	160	∞	3	—0,05	—0,05	0,5	80
7	0,0628	0,5	8	400	160	40	3	0	0	0,5	55
8	0,0628	0,5	8	400	160	∞	3	0	0	0,25	125
9	0,0628	0,5	8	400	160	∞	3	0	0	0,75	80
10	6,28	10	2	2000	200	∞	3	0	0	0,5	400
11	6,28	8	3,12	2500	391	∞	3	0	0	0,5	500
12	6,28	6	5,56	3340	928	∞	3	0	0	0,5	660

The flow of the jet is studied as a Cauchy problem with the conditions formulated below at x = 0. The stream surfaces $y = h_n(x)$, $n = 1, \ldots, N$, are introduced for the numerical solution; in addition, $h_1 = 0$, $h_M = H_1$, $h_N = H_2$, and the values of the velocity components on them $u_n(x) = U_1(x, h_n(x))$, $w_n(x) = W_1(x, h_n(x))$, $n = 1, \ldots, M$, $u_n(x) = U_2(x, h_n(x))$, $w_n(x) = W_2(x, h_n(x))$, $n = M + 1, \ldots, N$. For the variables h_n, u_n, and w_n we obtain from (1.1)-(1.6) a system of ordinary differential equations [3, 4]:

$$\frac{dh_1}{dx} = 0, \quad \frac{dh_n}{dx} = \frac{1}{2h_n u_n + h_{n-1}(u_{n-1} - u_n)} \Big\{ [h_n(u_n - u_{n-1}) +$$

$$+ 2h_{n-1}u_{n-1}] \frac{dh_{n-1}}{dx} - (h_n - h_{n-1}) \left(h_n \frac{du_n}{dx} + h_{n-1} \frac{du_{n-1}}{dx} \right) \Big\}, \quad n = 2, \ldots, N,$$

$$\frac{du_n}{dx} = \frac{1}{u_n} \left[\frac{1}{\mathrm{Fr}} + \frac{1}{\mathrm{We}} \left(\frac{1}{h_M^2} \frac{dh_M}{dx} + \frac{\gamma}{h_N^2} \frac{dh_N}{dx} \right) + T_{1n} \right], \quad n = 1, \ldots, M,$$

$$\frac{du_n}{dx} = \frac{1}{u_n} \left(\frac{1}{\mathrm{Fr}} + \frac{\gamma}{\lambda \mathrm{We} h_N^2} \frac{dh_N}{dx} + \alpha T_{2n} \right), \quad n = M + 1, \ldots, N, \tag{1.7}$$

$$\frac{dw_n}{dx} = \alpha_k S_{kn} - \frac{w_n}{h_n} \frac{dh_n}{dx}, \quad S_{kn} = \frac{1}{u_n \mathrm{Re}} \left(\left. \frac{\partial^2 W_k}{\partial y^2} \right|_{y=h_n} + \frac{1}{h_n} \left. \frac{\partial W_k}{\partial y} \right|_{y=h_n} - \frac{w_n}{h_n^2} \right),$$

$$T_{kn} = \frac{1}{\mathrm{Re}} \left(\left. \frac{\partial^2 U_k}{\partial y^2} \right|_{y=h_n} + \frac{1}{h_n} \left. \frac{\partial U_k}{\partial y} \right|_{y=h_n} \right), \quad k = 1: \; n = 2, \ldots, M,$$

$$k = 2: \; n = M + 1, \ldots, N, \quad T_{11} = \frac{2}{\mathrm{Re}} \left. \frac{\partial^2 U_1}{\partial y^2} \right|_{y=h_1}, \quad \frac{dw_1}{dx} = 0,$$

where the expression for T_{11} was obtained by expanding the function U_1 in a Taylor series near the axis of the jet.

The tau approximation [6] using shifted Chebyshev polynomials of the first kind $\varphi_k(\eta)$, defined by the formulas [7] $\varphi_1 = 1$, $\varphi_2 = 2\eta - 1$, $\varphi_k = 2\varphi_2\varphi_{k-1} - \varphi_{k-2}$, $k = 3, 4, \ldots$, is employed, as done in [5], for calculating the y derivatives in (1.7). In so doing, two approximating functions are constructed for the components of the velocity, for example, the axial component,

$$\Psi_1 = \sum_{k=1}^{M+2} a_k \varphi_k \left(\frac{y}{h_M} \right), \quad \Psi_2 = \sum_{k=1}^{N-M+3} a_{M+2+k} \varphi_k \left(\frac{y - h_M}{h_N - h_M} \right),$$

the expansion coefficients in which a_k $(k = 1, \ldots, N + 5)$ are the solutions of the system of linear algebraic equations

$$\sum_{k=1}^{M+2} a_k \varphi_k \left(\frac{h_n}{h_M} \right) = u_n, \quad n = 1, \ldots, M, \quad \sum_{k=1}^{N-M+3} a_{M+2+k} \varphi_k \left(\frac{h_n - h_M}{h_N - h_M} \right) = u_n, \tag{1.8}$$

$$n = M, \ldots, N;$$

$$\sum_{k=1}^{M+2} a_k \varphi_k'(0) = 0, \quad \sum_{k=1}^{N-M+3} a_{M+2+k} \varphi_k'(1) = 0; \tag{1.9}$$

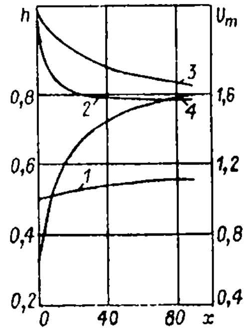

Fig. 1

$$\frac{1}{h_M}\sum_{k=1}^{M+2} a_k\varphi_k'(1) - \frac{\alpha\lambda}{h_N - h_M}\sum_{k=1}^{N-M+3} a_{M+2+k}\varphi_k'(0) = 0; \quad (1.10)$$

$$\frac{1}{h_M^2}\sum_{k=1}^{M+2} a_k\left[\varphi_k''(1) + \varphi_k'(1)\right] - \frac{\alpha}{h_N - h_M}\sum_{k=1}^{N-M+3} a_{M+2+k}\times$$
$$\times\left[\frac{1}{h_N - h_M}\varphi_k''(0) + \frac{1}{h_M}\varphi_k'(0)\right] + \frac{\mathrm{Re}}{\mathrm{We}}\left(\frac{1}{h_M^2}\frac{dh_M}{dx} + \frac{\lambda - 1}{\lambda}\frac{\gamma}{h_N^2}\frac{dh_N}{dx}\right) = 0, \quad (1.11)$$

where (1.8) expresses the fact that the functions Ψ_1, Ψ_2 equal the values of the axial velocity on the stream surfaces; (1.9) is the approximation of the boundary conditions on the axis and on the surface of the jet; (1.10) is the approximation of the continuity of the tangential stress on the interface; and, (1.11) expresses the fact that the equations of motion of the liquids at the interface must match.

After the values of a_k (k = 1, ..., N + 5) are determined the coefficients in the series expansion of the y derivatives of the functions Ψ_1, Ψ_2 in Chebyshev polynomials can be calculated and the values of these derivatives on the stream surfaces can be determined for substituting into (1.7) [5].

To implement this algorithm it is necessary to calculate the values of dh_M/dx, dh_N/dx, appearing in (1.8)-(1.11). To this end we obtain from (1.7) a system of linear algebraic equations of the form

$$b_n\frac{dh_{n-1}}{dx} + \frac{dh_n}{dx} + c_n\frac{dh_M}{dx} + d_n\frac{dh_N}{dx} = e_n, \quad n = 1, \ldots, N, \quad (1.12)$$

$b_1 = 0$; $c_k = 0$ (k = M + 1, ..., N); the coefficients in (1.12) depend on the values of the y derivatives of the axial component on the stream surfaces. A system of two implicit linear algebraic equations for determining dh_M/dx, dh_N/dx follows from (1.8)-(1.12). We note that (1.12) is solved by the sweep method [8].

The difference between the algorithm for calculating the derivatives of the azimuthal velocity and the algorithm described above lies in the use of the (M + 1)-st polynomial to approximate the velocity of the interior liquid; in addition, the condition of matching of the equations at the interface does not contain the values of dh_M/dx, dh_N/dx.

The equations (1.7) must be supplemented by the initial conditions

$$h_n(0) = \frac{n-1}{M-1}H_1(0), \quad u_n(0) = U_{10}(h_n), \quad w_n(0) = W_{10}(h_n), \quad n = 1, \ldots, M,$$
$$h_n(0) = H_1(0) + [1 - H_1(0)]\frac{n-M}{N-M}, \quad u_n(0) = U_{20}(h_n), \quad w_n(0) = W_{20}(h_n),$$
$$n = M+1, \ldots, N,$$

where U_{10}, U_{20}, W_{10}, W_{20} are fixed functions; $H_1(0)$ is the starting radius of the interface. The equations (1.7) are integrated by the Adams—Bashfort method with second-order accuracy [6].

2. Because of the action of viscous forces, a nearly constant profile of the axial velocity is formed as x increases, irrespective of the form of the initial conditions; for a spiral flow the dependence of the azimuthal velocity on the radius approaches a linear dependence. In the cases Fr = ∞ and Fr ≠ ∞, $\sqrt{q_1} = \lambda/[\gamma(1 - \lambda)]$ the problem (1.1)-(1.6) has the solution

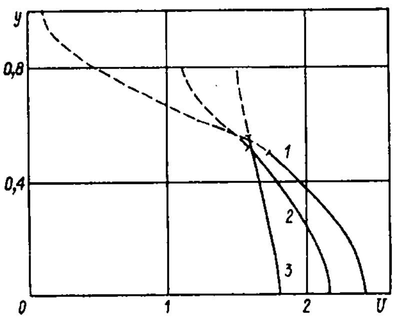

Fig. 2

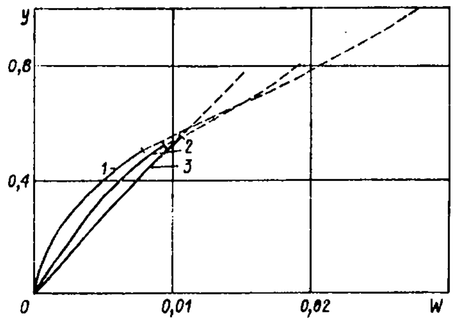

Fig. 3

$$H_1 = H_2\sqrt{q_1},\quad U_i = \frac{1}{H_2^2},\quad V_i = \frac{y}{H_2^3}\frac{dH_2}{dx},\quad W_i = \omega y,\quad i = 1, 2. \tag{2.1}$$

Here $\omega = \omega_a H_{2a}^2/H_2^2$; $q_1 = 2\int\limits_0^{H_1(0)} yU_{10}dy$; H_{2a}, ω_a are the radius of the jet and its angular rotational velocity as a rigid body for some value $x = x_a$; the solution (2.1) is valid for $x \geqslant x_a$. The dependence $H_2(x)$ is determined from the condition

$$\frac{1}{2}\left(\frac{1}{H_2^4} - \frac{1}{H_{2a}^4}\right) + \frac{\gamma}{\lambda\,\mathrm{We}}\left(\frac{1}{H_2} - \frac{1}{H_{2a}}\right) - \frac{x - x_a}{\mathrm{Fr}} = 0.$$

The length of the section of formation of the uniform solution depends on the parameters Re, We, Fr, α, λ, γ, the flow rate q_1, and the starting profiles of the velocity components. In studying the dependence of x_a on the parameters we studied the case

$$U_{i0} = 1.5\beta_i(y^4 - 2y^2) + \varepsilon_i,\ i = 1, 2,\ \varepsilon_1 = \varepsilon_2 + 1.5\beta_2 h^2(h^2 - 2)(1 - \alpha\lambda),$$

$$\beta_1 = \beta_2\alpha\lambda,\quad \beta_2 = \frac{\varepsilon_2 - 1}{1 - (1 - \alpha\lambda)\,h^4\,(h^2 - 1.5)},$$

$$W_{10} = t_1 y + 2\alpha t_2\,[4(\lambda - 1)\,h + 3 - 4\lambda]\,y^2 + \frac{\alpha t_2}{h}\,[(4 - 3\lambda)\,h + 3(\lambda - 1)]\,y^3,$$

$$W_{20} = \Big\{t_1 + 2t_2\,[4\alpha(\lambda - 1)\,h + (3 - 4\lambda)\,\alpha + 1]\,h + t_2\Big[\frac{3\alpha(\lambda - 1)}{h} + {}$$
$$+ \alpha(4 - 3\lambda) - 1\Big]h^2\Big\}\,y - 2t_2y^2 + t_2y^3,$$

where $h = H_1(0)$; the form of the starting axial velocity is determined by the parameter ε_2 and that of the azimuthal velocity is determined by the parameters t_1, t_2. Table 1 gives for a jet, whose interior liquid is water and the exterior liquid is benzene and for which $\alpha = 0.709$, $\lambda = 0.752$, $\gamma = 1.5$, the values of the parameters and the computed corresponding lengths of the section of formation of the uniform solution x_a. As the criterion for calculating x_a we chose the condition $|1 - f| < 0.05$. Here $f = u_n/U_{1m}$ $(n = 1, \ldots, M)$, $w_n/(\omega_1 h_n)$ $(n = 2, \ldots, M)$, u_n/U_{2m}, $w_n/(\omega_2 h_n)$ $(n = M, \ldots, N)$, U_{min}/U_{max}, $\omega_{min}/\omega_{max}$, $U_{1m} = \frac{2}{H_1^2}\int\limits_0^{H_1} yU_1dy$, $U_{2m} = \frac{2}{H_2^2 - H_1^2}\int\limits_{H_1}^{H_2} yU_2dy$ are the average values of the axial velocity for the interior and exterior liquids, $\omega_1 = w_M/h_M$, $\omega_2 = w_N/h_N$ are the angular rotational velocities of the liquid on the interface and on the surface of the jet, U_{min} and U_{max}, ω_{min} and ω_{max} are the minimum and maximum values of U_{1m}, U_{2m} and ω_1, ω_2, respectively. The value of x_a was calculated with an accuracy of $\Delta x_a = 5$ for calculations 1, 2, 4, 6-9 and $\Delta x_a = 20$ for calculations 3, 5, and 10-12.

Figure 1 shows for calculation 5 the dependences H_1, H_2, U_{1m}, U_{2m}, which correspond to the curves 1-4. For the same calculation Figs. 2 and 3 show the profiles of the velocity components; in addition, Fig. 2 shows the axial component for x = 0, 20, and 60 (lines 1-3); Fig. 3 shows the azimuthal component for x = 0, 20, and 90 (curves 1-3); in both cases the profiles in the interior liquid are denoted by a solid line while the profiles in the exterior liquid are denoted by the broken line.

Thus the proposed method for calculating flows of composite jets permits finding, aside from the value of x_a, the characteristics of the flow in intermediate sections and their limiting values, making it possible to study later the stability of the given flow.

In conclusion we note that the method can be extended to the case of a multilayer jet, but the larger number of implicit linear differential equations in this case for determining the x derivatives as a function of the ordinates of the interface surfaces makes it necessary to execute the approximation algorithm repeatedly for a fixed value of x; this substantially increases the computing time.

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INVESTIGATION OF NATURAL CONVECTION AND CONVECTION STIMULATED BY LOCAL IRRADIATION IN A THIN LAYER OF EVAPORATING LIQUID

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Capillary convection in low-viscosity liquids is observed with surface-tension differentials of the order of 0.1 mN/m. It appears in many technological processes. Convection leading to the formation of a relief on the boundary of the interface arises in a layer of drying paint and varnish or glassy enamel as well as accompanying extraction in liquid—liquid systems or rectification of multicomponent mixtures [1-4]. Fluctuational surface-tension gradients initiate convection.

The high sensitivity of liquids to shear stresses has been employed to solve technical problems, such as separation of impurities [5], obtaining relief photographic images [6, 7], deposition of matter at a fixed location of a substrate [8], or surface doping of metals [9]. The technical solutions listed are based on capillary convection, controlled by the thermal action of radiation [10, 11]. In spite of the wide range of possible applications of forced capillary convection virtually no quantitative data on convection under the action of radiation and its comparison with spontaneous convective processes have been published. The results of a study of capillary-convective instability of a layer of liquid in the regime of natural evaporation and under conditions of local heating by low-power laser radiation are presented below.

1. Materials and Methods. Convection in solutions of crystal violet dye in polar organic solvents was studied (see Table 1). The dye contrasted the image of the relief on the surface of the layer and simultaneously functioned as a strain-active and light-absorbing additive. Thermal capillary convection was induced by the action of a Gaussian beam of

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